WP4 - Flexibility measures in different climate zones & markets

D4.1 GREY BOX MODELS OF THE DEMONSTRATION CASES

Seyed Shahabaldin Tohidi - DTU
Behrouz Eslami Mossallam - TNO
Davide Cali - DTU

31.03.2022/M27
1. Revision Log:
This deliverable contains original unpublished work except where clearly indicated otherwise. Acknowledgement of previously published work has been properly cited.

PROJECT ACRONYM ......... syn.ikia
PROJECT NUMBER .......... 869918
PROJECT TITLE ............. Sustainable Plus Energy Neighbourhoods
WEBSITE ..................... www.synikia.eu

2. Technical References

<table>
<thead>
<tr>
<th>Deliverable (number)</th>
<th>D4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deliverable Name</td>
<td>Grey box models of the demonstration cases</td>
</tr>
<tr>
<td>Work Package</td>
<td>WP4</td>
</tr>
<tr>
<td>Task number and Title</td>
<td>Task 4.1, Grey box models of the demonstration case</td>
</tr>
<tr>
<td>Dissemination Level</td>
<td>PU</td>
</tr>
<tr>
<td>Date of Delivery</td>
<td>31.03.2022</td>
</tr>
<tr>
<td>Lead Beneficiary</td>
<td>Seyed Shahabaldin Tohidi – DTU Davide Cali – DTU</td>
</tr>
<tr>
<td>Contributors</td>
<td>Behrouz Eslami Mossallam - TNO</td>
</tr>
<tr>
<td>Reviewers</td>
<td>Ruud van der Linden</td>
</tr>
<tr>
<td>Status</td>
<td>Completed</td>
</tr>
</tbody>
</table>

Document history

<table>
<thead>
<tr>
<th>Version</th>
<th>Date</th>
<th>Authors/Institutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>V0</td>
<td>17-03-2022</td>
<td>Seyed Shahabaldin Tohidi – DTU Davide Cali – DTU Behrouz Eslami Mossallam - TNO</td>
</tr>
<tr>
<td>V1</td>
<td>23-03-2022</td>
<td>Ruud van der Linden (review)</td>
</tr>
<tr>
<td>V2</td>
<td>28.03.2022</td>
<td>Seyed Shahabaldin Tohidi, Davide Cali, Behrouz Eslami</td>
</tr>
</tbody>
</table>
3. Executive Summary

This report is a short-written description of deliverable 4.1, D4.1: the deliverable itself consists of the Grey-box models of the 4 demo sites in syn.ikia project.
Table of contents

1. REVISION LOG: ................................................................. 1

2. TECHNICAL REFERENCES ...................................................... 1

3. EXECUTIVE SUMMARY .......................................................... 2

4. ROLES AND RESPONSIBILITIES ............................................. 4

5. INTRODUCTION .................................................................. 4

6. SPANISH DEMO GREY-BOX .................................................. 5

7. AUSTRIAN DEMO GREY-BOX .................................................. 7

8. NORWEGIAN DEMO GREY-BOX ............................................. 9

9. DUTCH DEMO GREY-BOX ..................................................... 11

10. DUTCH DEMO SIMPLIFIED WHITE-BOX ............................. 13

11. OUTLOOK ...................................................................... 15

12. REFERENCES ..................................................................... 16

13. APPENDIX A – GLOSSARY OF TERMS: .............................. 16
4. Roles and Responsibilities

<table>
<thead>
<tr>
<th>Name</th>
<th>Role</th>
<th>Responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTU</td>
<td>Task 4.1 leader, coordinator of deliverable contents, contributor</td>
<td>Generating grey-box models of the four demo cases, evaluation of different models for each demo and correct model selection, coordinating the activities of the partners, design simulation scenarios for the white box models, so that the results from white box models can be used to fit the grey-box models.</td>
</tr>
<tr>
<td>TNO</td>
<td>Contributor</td>
<td>Generating specific simulation results from the white box models from the NL demo, generation of a digital twin, that can be used instead of a grey-box model in the NL case.</td>
</tr>
<tr>
<td>NTNU</td>
<td>Contributor</td>
<td>Generating specific simulation results from the white box models from the Norwegian demo</td>
</tr>
<tr>
<td>IREC</td>
<td>Contributor</td>
<td>Generating specific simulation results from the white box models from the Spanish demo</td>
</tr>
<tr>
<td>ABUD</td>
<td>Contributor</td>
<td>Generating specific simulation results from the white box models from the Austrian demo</td>
</tr>
</tbody>
</table>

5. Introduction

High quality models able to predict the future evolution of thermal dynamics, are required for the advanced model-based control implementation. In particular, these models should capture nonlinear behaviours of the thermal dynamics in the presence of process noise due to approximation errors or unmodelled inputs and measurement noise due to imperfect measurements.

The grey-box (GB) models, which consist of a set of stochastic differential equations (SDEs) that describe the dynamics of the system in continuous time and a set of discrete time measurement equations, allow incorporation of prior physical knowledge and utilization of statistical methods for parameter estimation. The physically meaningful parameters made these models a proper choice for control purposes.

Physical knowledge of buildings and the information embedded in the collected data from the buildings are two main requirements of establishing a grey-box model. The physical knowledge can be formulated by a set of first-order stochastic differential equations. Since the goal of finding a grey-box model is to design a controller, such as model predictive controller (MPC), it is meaningful to use the optimal simplified grey-box model which consists of a set of first-order linear stochastic differential equations.

In this mini-report, we describe briefly how we employed the generated data of white-box models for Spanish, Austrian, Norwegian and Dutch demos to estimate the parameters and find the best grey-box model for each demo. Statistical analyses were used to establish the model selection procedure (Bacher & Madsen, 2011) (Tohidi, Cali, Tamm, Ortiz, & Salom, 2022). A brief description of parameter estimation and model selection procedures are provided here:
Various electric circuits resembling the heat dynamics of buildings are considered and their dynamical equations are found. Then, a Kalman filter is applied to calculate the likelihood function, and an optimization problem is solved to maximize it. This can be done using the computer software CTSMR capable of calculating the maximum likelihood and estimating parameters of each model simultaneously (Juhl, Møller, & Madsen, 2016).

Once the parameters of each dynamical system were estimated, we proceeded with the selection of the best grey-box model per demo. To be mathematically rigorous, three statistical analyses were applied for model selection. These analyses are likelihood ratio test, Akaike’s and Bayesian criteria. Likelihood ratio test finds the significant improvement between each two models with different number of parameters. Different from Likelihood ratio test, Akaike’s and Bayesian criteria allocate a specific number to each model that simplifies the model selection decision making (Konishi & Kitagawa, 2008).

Simulation results demonstrate that the one-step ahead error of the selected model for each demo has white-noise-like properties which indicates that these are appropriate models to be used for the control purposes. Prediction capabilities of the selected models for each demo are illustrated using cross-validation.

### 6. Spanish demo Grey-Box

The best model, selected for one specific apartment of Spanish demo, is found to be the following two-state grey-box model:

\[
\begin{align*}
C_i dT_i &= \left( \Phi_h + A_w \Phi_s + \frac{T_e - T_i}{R_{ie}} \right) dt + \sigma_i d\omega_i \\
C_e dT_e &= \left( \frac{T_i - T_e}{R_{ie}} + \frac{T_a - T_e}{R_{ea}} \right) dt + \sigma_e d\omega_e \\
Y_k &= T_{i,k} + e_k,
\end{align*}
\]

where \(T_i\) and \(T_e\) are the states, that represent the interior and environment temperatures, respectively, \(C_i\) and \(C_e\) are the thermal capacities of interior and envelope, respectively, and \(R_{ea}\) and \(R_{ie}\) are the thermal resistances between ambient and envelope, and envelope and interior. Also, \(T_a\) is the ambient temperature, \(\Phi_h\) is the total heat input, \(\Phi_s\) is the solar irradiance, \(A_w\) is the effective window area and \(Y_k\) is the measured interior temperature. To represent the stochastic behaviour of the heat dynamics, we introduce \(\omega_i\) and \(\omega_e\) as standard Wiener processes, where \(\sigma_i^2\) and \(\sigma_e^2\) are the incremental variances of the Wiener processes. The deterministic part of the model is physically meaningful and can be considered as the following RC circuit.

![Figure 1. RC circuit based on the deterministic thermal dynamics of Spanish demo.](image)

By comparing the measured and the modelled temperatures, \(Y_k\) and \(T_i\), it is seen that the error, \(e_k\), represents similar properties as the ones for white noise. Figure 2 demonstrates that the error is almost uncorrelated in time, its spectrum is uniformly spread across the frequencies, and the cumulative periodogram is close to the straight line. These are properties of a white noise (Ljung, 1998) (Madsen, 2007).
Figure 2. Statistical properties of one-step-ahead error of the selected model.

Figure 3 illustrates the 24-hours and 48-hours prediction capabilities of the selected model.
7. Austrian demo Grey-Box

The best model, selected for one specific apartment of Austrian demo, is found to be the following three-state grey-box model:

\[
\begin{align*}
C_i dT_i &= \left( -\frac{T_i - T_h}{R_{ih}} - \frac{T_i - T_e}{R_{ie}} + \frac{T_a - T_i}{R_{ia}} + A_w \Phi_s \right) dt + \sigma_i d\omega_i \\
C_h dT_h &= \left( \frac{T_i - T_h}{R_{ih}} + \Phi_h \right) dt + \sigma_h d\omega_h \\
C_e dT_e &= \left( \frac{T_i - T_e}{R_{ie}} + \frac{T_a - T_e}{R_{ea}} \right) dt + \sigma_e d\omega_e \\
Y_k &= T_{i,k} + e_k,
\end{align*}
\]

where \(T_i, T_h\) and \(T_e\) are the states of interior, heater and envelope temperatures, respectively. Also \(C_i, C_h\) and \(C_e\) are the thermal capacities of interior, heater and envelope, respectively. \(R_{eq}, R_{ie}\) and \(R_{ia}\) are the thermal resistances between ambient and envelope, envelope and interior, and ambient and interior, respectively, and \(R_{ih}\) is the heater resistance. Also, \(T_a\) is the ambient temperature, \(\Phi_h\) is the total heat input, \(\Phi_s\) is the solar irradiance, \(A_w\) is the effective window area and \(Y_k\) is the measured interior temperature. To represent the stochastic behaviour of the heat dynamics, we introduce \(\omega_i, \omega_h\) and \(\omega_e\) as standard Wiener processes, where \(\sigma_i^2, \sigma_h^2\) and \(\sigma_e^2\) are the incremental variances of the Wiener processes. The deterministic part of the model is physically meaningful and can be considered as the following RC circuit.
By comparing the measured and the modelled temperatures, $Y_k$ and $T_i$, it is seen that the error, $e_k$, represents similar properties as the ones for white noise. Figure 5 demonstrates that the error is almost uncorrelated in time, its spectrum is uniformly spread across the frequencies, and the cumulative periodogram is close to the straight line. These are properties of a white noise (Ljung, 1998) (Madsen, 2007).
Figure 6 illustrates the 24-hours and 48-hours prediction capabilities of the selected model.

Figure 6. 24-hours (top) and 48-hours (bottom) ahead interior temperature prediction.

8. Norwegian demo Grey-Box

The best model, selected for one specific apartment of Norwegian demo, is found to be the following three-state grey-box model:

\[
\begin{align*}
C_i dT_i &= \left( \frac{-T_i + T_h}{R_{ih}} + \frac{-T_i + T_e}{R_{ie}} + A_w \Phi_s \right) dt + \sigma_i d\omega_i \\
C_h dT_h &= \left( \frac{T_i - T_h}{R_{ih}} + \Phi_h \right) dt + \sigma_h d\omega_h \\
C_e dT_e &= \left( \frac{T_i - T_e}{R_{ie}} + \frac{T_a - T_e}{R_{ea}} \right) dt + \sigma_e d\omega_e \\
Y_k &= T_{i,k} + e_k,
\end{align*}
\]

where \( T_i \), \( T_h \) and \( T_e \) are the states, that represent the interior, heater and envelope temperatures, respectively, \( C_i \), \( C_h \) and \( C_e \) are the thermal capacities of interior, heater and envelope, respectively, and \( R_{ea} \) and \( R_{ie} \) are the thermal resistances between ambient and envelope, and envelope and interior. Also, \( R_{ih} \) is the heater thermal resistance, and \( T_a \) is the ambient temperature, \( \Phi_h \) is the total heat input, \( \Phi_s \) is the solar irradiance, \( A_w \) is the effective window area and \( Y_k \) is the measured interior temperature. To represent the stochastic behaviour of the heat dynamics, we introduce \( \omega_i \), \( \omega_h \) and \( \omega_e \) as standard Wiener processes, where \( \sigma_i^2 \), \( \sigma_h^2 \) and \( \sigma_e^2 \) are the incremental variances of the Wiener processes. The deterministic part of the model is physically meaningful and can be considered as the following RC circuit.
By comparing the measured and the modelled temperatures, $Y_k$ and $T_i$, it is seen that the error, $e_k$, represents similar properties as the ones for white noise. Figure 8 demonstrates that the error is almost uncorrelated in time, its spectrum is uniformly spread across the frequencies, and the cumulative periodogram is close to the straight line. These are properties of a white noise (Ljung, 1998) (Madsen, 2007).

Figure 9 illustrates the 24-hours and 48-hours prediction capabilities of the selected model.
9. Dutch demo Grey-Box

The best model, selected for one specific apartment of Dutch demo, is found to be the following two-state grey-box model:

\[
\begin{align*}
C_i dT_i &= \left( \frac{-T_i + T_h}{R_{ih}} + \frac{-T_i + T_a}{R_{ia}} + A_{w,W} \Phi_{s,W} + A_{w,E} \Phi_{s,E} \right) dt + \sigma_i d\omega_i \\
C_h dT_h &= \left( \frac{T_i - T_h}{R_{ih}} + \Phi_h \right) dt + \sigma_h d\omega_h \\
Y_k &= T_{ik} + \epsilon_k,
\end{align*}
\]

where \( T_i \) and \( T_h \) are the states, that represent the interior and the heater temperatures, respectively, \( C_i \) and \( C_h \) are the thermal capacities of interior and heater, and \( R_{ia} \) and \( R_{ih} \) are the envelope and the heater temperatures. Also, \( R_{ih} \) is the heater resistance, and \( T_a \) is the ambient temperature, \( \Phi_h \) is the total heat input, \( \Phi_{s,W} \) and \( \Phi_{s,E} \) are the solar irradiances from West and East, \( A_{w,W} \) and \( A_{w,E} \) are the west and east sides effective window areas and \( Y_k \) is the measured interior temperature. It is noted that the Dutch demo uses both heating and cooling, therefore positive and negative values of \( \Phi_h \) are related to the heating and cooling processes, respectively. To represent the stochastic behaviour of the heat dynamics, we introduce \( \omega_i, \omega_h \) and \( \omega_e \) as standard Wiener processes, where \( \sigma_i^2, \sigma_h^2 \) and \( \sigma_e^2 \) are the incremental variances of the Wiener processes. The deterministic part of the model is physically meaningful and can be considered as the following RC circuit.
By comparing the measured and the modelled temperatures, $Y_k$ and $T_i$, it is seen that the error, $e_k$, represents similar properties as the ones for white noise. Figure 11 demonstrates that the error is almost uncorrelated in time, its spectrum is uniformly spread across the frequencies, and the cumulative periodogram is close to the straight line. These are properties of a white noise (Ljung, 1998) (Madsen, 2007).

Figure 11. Statistical properties of one-step-ahead error of the selected model.

Figure 12 illustrates the 24-hours and 48-hours prediction capabilities of the selected model.
10. Dutch demo simplified White-Box

The Dutch demo is modelled in SirinE, a hybrid predictive digital twin for buildings. SirinE consists of a physical building model which solves the heat flow balance equations, and a data-driven occupant model which models the interaction of the occupants with the building components (e.g., thermostats, windows, electric appliances, etc.) and includes the effect of occupants’ actions in the heat flow balance equations.

A multizone model for the Dutch Demo is constructed, where each room is considered a thermal zone. Each zone $z_i$ is represented by a temperature node $T_{zi}$ in the heat network. Each physical layer of boundary surfaces (i.e., walls, floor, ceiling and roofs) constitutes a temperature node in the heat network. For $k$th layer of the $j$th boundary surface $S_{jk}$ ($k=1$ corresponds to the innermost layer, $k=n$ to the outermost one), a temperature node $T_{S_{jk}}$ is added to the heat network. In addition, all boundaries (outdoor environment, ground etc.) are represented by a temperature node. The heat flow balance equations can be summarized as follows:

- For the zone $z_i$:
  \[
  C_{zi} \frac{\partial}{\partial t} T_{zi} = \sum_{S_{j} \in z_i} A_j h_{\text{int}}^{\text{surf}} (T_{S_{j,n}} - T_{zi}) + Q_{zi,\text{vent}} + Q_{zi,\text{int}} + Q_{zi,\text{sol}} + Q_{zi,\text{heat/cool}}
  \]

- For the innermost surface layer $S_{j,1}$ of the boundary surface $S_{j}$ which is in direct contact with the zone $z_i$:
  \[
  C_{S_{j,1}} \frac{\partial}{\partial t} T_{S_{j,1}} = A_j h_{\text{int}}^{\text{surf}} (T_{zi} - T_{S_{j,1}}) + A_j h_{2,1}^{(j)} (T_{S_{j,2}} - T_{S_{j,1}})
  \]
For the internal layers of the surface $S_j$

$$C_{S_{j,k}} \frac{\partial}{\partial t} T_{S_{j,k}} = A_{j} h_{k,k-1}^{(j)} (T_{S_{j,k-1}} - T_{S_{j,k}}) + A_{j} h_{k+1,k}^{(j)} (T_{S_{j,k+1}} - T_{S_{j,k}})$$

For the outermost surface layer $S_{j,n}$ in contact with the outside environment:

$$C_{S_{j,n}} \frac{\partial}{\partial t} T_{S_{j,n}} = A_{j} h_{n,n-1}^{(j)} (T_{S_{j,n-1}} - T_{S_{j,n}}) + A_{j} h_{\text{surf}}^{\text{ext-conv}} (T_{\text{out}} - T_{S_{j,n}}) + A_{j} h_{\text{surf}}^{\text{ext-rad}} (T_{\text{out}} - T_{S_{j,n}}) + A_{j} F_{\text{sky}} h_{\text{surf}}^{\text{ext-rad}} (T_{\text{Sky}} - T_{\text{out}}) + Q_{S_{j,\text{sol}}}
$$

The parameters in the above equations are defined as:

- $C_{z_{i}}$: thermal mass of zone $z_{i}$.
- $C_{S_{j,k}}$: thermal mass of surface layer $S_{j,k}$ of the boundary surface $S_{j}$.
- $A_{j}$: the area of the boundary surface $S_{j}$.
- $h_{\text{int}}^{\text{surf}}$: the internal surface heat transmission coefficient, including both the convective and radiative transmissions.
- $h_{\text{ext-conv}}^{\text{surf}}$: the external convective surface heat transmission coefficient.
- $h_{\text{ext-rad}}^{\text{surf}}$: the external radiative surface heat transmission coefficient.
- $h_{k,k-1}^{(j)}$: conductive heat transmission coefficient between the $k$th and $(k-1)$th layers of the boundary surface $S_{j}$.
- $T_{\text{out}}$: outdoor temperature.
- $T_{\text{Sky}}$: apparent sky temperature.
- $F_{\text{sky}}$: view factor to the sky for the boundary surface $S_{j}$.
- $Q_{z_{i,\text{vent}}}$: ventilation heat flow for the zone $z_{i}$. In SirinE this is calculated by solving the steady-state airflow balance equations.
- $Q_{z_{i,\text{int}}}$: internal heat flow for the zone $z_{i}$ due to occupants and house appliances.
- $Q_{z_{i,\text{sol}}}$: solar heat flow for the zone $z_{i}$ via the windows.
- $Q_{S_{j,\text{sol}}}$: absorbed solar power by the external boundary surface $S_{j}$.
- $Q_{z_{i,\text{heat/cool}}}$: heating or cooling flow delivered to zone $z_{i}$ via the floor heating system. In SirinE this is calculated by modelling a plant loop system, which includes a heat pump, a source-side pump, a load-side pump and a ground heat exchanger and solving the steady-state flow and heat flow balance equations.

In addition to the building model, a combined tank and heat pump model is used to simulate the production of the domestic hot water (DHW). The model solves the heat flow balance equations between the layers of hot water inside the tank taking into account the heat stratification in the water volume.

The simulation is performed using a heat pump control comparable to the real installation control (which uses an on-off controlled heat pump for both heating the building as for heating of domestic water).

Figures 13 and 14 show the simulated temperature profiles as well as the electric power consumption by the heat pump during a whole year for the Dutch demo.
11. Outlook

This report provides a brief discussion and simulation results of the grey-box models estimated for Spanish, Austrian, Norwegian and Dutch demos. To find the best model capable of representing the thermal dynamics
of each demo, data generated by white-box modes has been used. The selected grey-box model can then be utilized for the model predictive control design purposes of task 4.5. In the task 4.2 the grey-box models will be finetuned according to different occupant behaviours.

In addition to the grey-box models, a simplified white-box model for the Dutch demo has also been presented. Compared to the grey-box model, a (simplified) white-box model requires a deeper knowledge of the buildings’ structure and geometry: In white-box models, most parameters are estimated based on previous knowledge, not based on measurements and telemetry data from buildings.

Both white-box and grey-box models can be used for control purposes, e.g., in a model predictive control approach, as a stand-alone, or even be combined: grey-box models are typically much faster than white-box models due to their smaller dimension. White-box models, on the other hand, are more accurate.

12. References


13. Appendix A – Glossary of Terms:

**GB**: grey-box

**MPC**: model predictive controller

**SDE**: stochastic differential equations
“This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 869918.”